(go to the Problem Statement)

The geometry of the problem is illustrated in Figure 1. A cylindrical coordinate system is used in order to take advantage of the symmetry of the problem. On the basis of the assumptions and symmetry we may immediately conclude: $v_z = 0$ and $v_\theta = 0$. There is no θ or z dependence of any variables. We are thus interested in determining within the polymer melt the variables: $v_r = v_r(r,t)$ and $P = P(r,t)$.

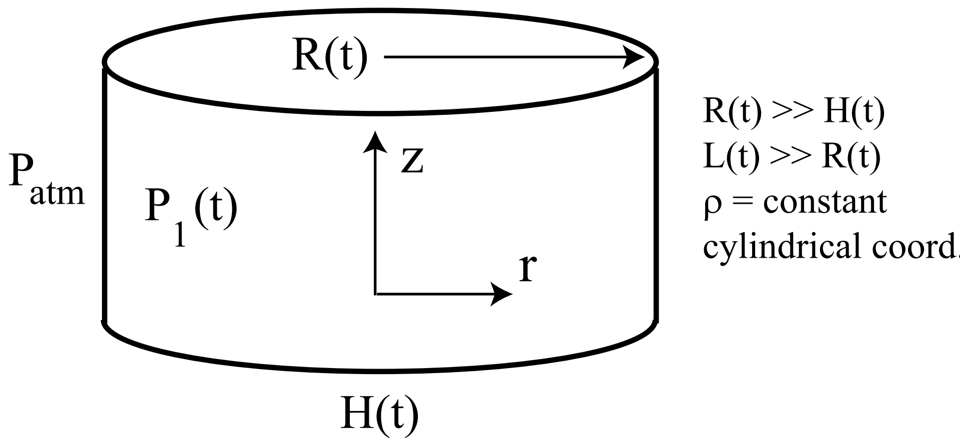

Figure 1: Sketch of problem geometry.

A constant inflation velocity means that: $\frac{dR(t)}{dt} = V_0 = constant$, which can be integrated to obtain: $R = R_0 + V_0 t$. Since the polymer is incompressible, the total volume of the cylindrical shell must remain constant. Using $H(t) \ll R(t)$ we may estimate: $Volume \cong 2\pi R_0 H_0 L = 2\pi R(t)H(t)L$, yielding

$$H(t) = H_0 \frac{R_0}{R_0 + V_0 t} \,.$$

The equation of continuity in cylindrical coordinates for an incompressible fluid with zero θ and z velocities reduces to:

$$\frac{\partial}{\partial r}(r v_r) + \frac{1}{r} = 0 \,.$$

This can be immediately integrated to obtain: $v_r = \frac{C(t)}{r}$ for some function $C(t)$. The boundary condition at the inner radius is:

$$R(t) v_r|_{R(t)} = R(t)\frac{dR(t)}{dt} = R(t)V_0 \,.$$

Application of this boundary condition yields:

$$v_r = V_0 \frac{R_0 + V_0 t}{r} . \tag{1}$$

The velocity gradient tensor for the current flow is:

$$\underline{\nabla}\,\underline{\mathbf{v}} = \begin{bmatrix} \frac{\partial v_r}{\partial r} & 0 & 0 \\ 0 & \frac{v_r}{r} & 0 \\ 0 & 0 & 0 \end{bmatrix} .$$

Substitution of the velocity field obtained above yields the rate of deformation tensor:

$$\underline{\underline{\dot{\gamma}}} = \begin{bmatrix} -2V_0 \frac{R_0 + V_0 t}{r^2} & 0 & 0 \\ 0 & 2V_0 \frac{R_0 + V_0 t}{r^2} & 0 \\ 0 & 0 & 0 \end{bmatrix} .$$

This is an extensional flow since the only nonzero components of the rate of deformation tensor are along the diagonal. The scalar shear rate is:

$$\dot{\gamma} = 2V_0 \frac{R_0 + V_0 t}{r^2} . \tag{2}$$

Assuming that the fluid is Newtonian with a constant viscosity μ and that the inertial terms are negligible, the equation of motion in cylindrical coordinates for a fluid of constant density reduces in this case to:

$$0 = -\frac{\partial P}{\partial r} + \mu \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} (r v_r) \right) .$$

However, from Equation 1 we know that $\frac{\partial}{\partial r}(r v_r) = 0$ and thus $\frac{\partial P}{\partial r} = 0$ and the pressure within the polymer is a function of t only, $P = P(t)$.

The constitutive equation for a Newtonian fluid is:

$$\underline{\underline{\pi}} = P \underline{\underline{\delta}} - \mu \underline{\underline{\dot{\gamma}}} .$$

Based on the results above, we can immediately write the total stress tensor:

$$\underline{\underline{\pi}} = \begin{bmatrix} P(t) + 2\mu V_0 \frac{R_0 + V_0 t}{r^2} & 0 & 0 \\ 0 & P(t) - 2\mu V_0 \frac{R_0 + V_0 t}{r^2} & 0 \\ 0 & 0 & 0 \end{bmatrix} .$$

Since interfacial tension and inertial effects are negligible the air pressures inside and outside the shell are balanced by the stresses within the polymer melt. We are thus interested in the stresses on the inside and outside surfaces of the expanding shell, which can be obtained by selecting the appropriate components of the total stress tensor $\underline{\underline{\pi}}$. At the inner surface we obtain:

$$P_1(t) = P(t) + 2\mu V_0 \frac{R_0 + V_0 t}{R^2(t)}$$

while at the outer surface

$$P_{atm}(t) = P(t) + 2\mu V_0 \frac{R_0 + V_0 t}{[R(t) + H(t)]^2} \,.$$

Subtracting these and recognizing that $H(t) \ll R(t)$ leads to:

$$P_1(t) - P_{atm} = \frac{4\mu V_0 H(t)}{R^2(t)} = \frac{4\mu V_0 H_0 R_0}{[R_0 + V_0 t]^3} \,.$$

The required blowing pressure as a function of time is plotted in Figure 2.

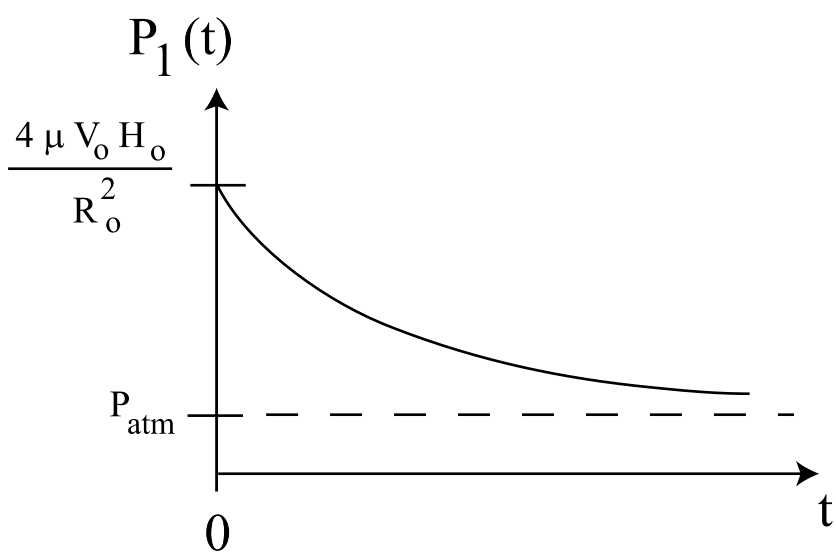

Figure 2: Blow pressure as a function of time.

For non-Newtonian fluids the shear rate still changes according to Equation 2 (Why?). Thus, the deformation rate decreases approximately as $1/R$.

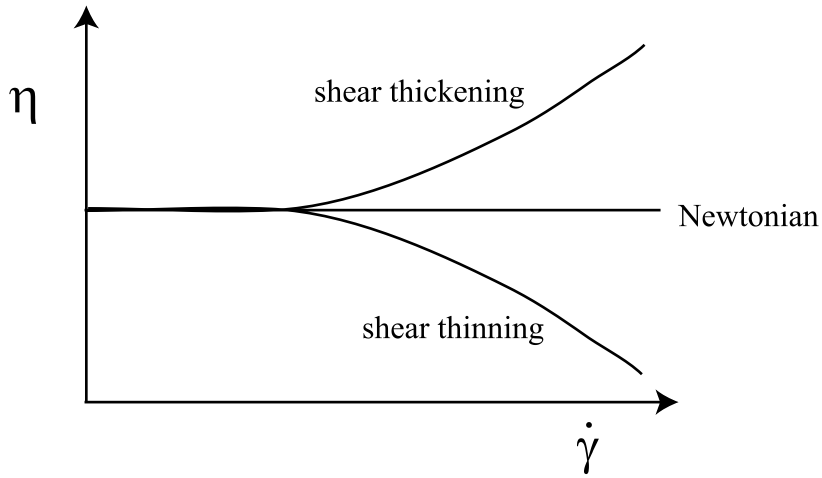

Figure 3: Qualitative behavior of three fluids with the same zero shear viscosity.

As a function of shear rate, three fluids of the same zero shear viscosity will qualitatively behave as illustrated in Figure 3. The resulting pressure effects are shown in Figure 4.

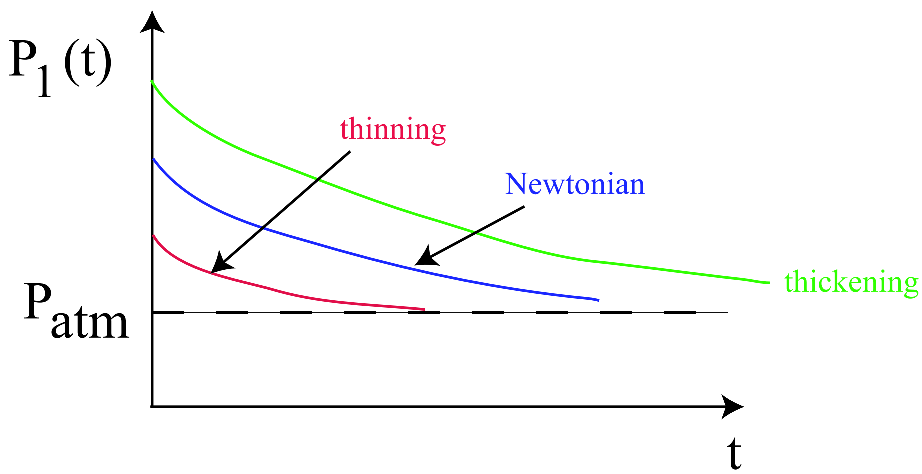

Figure 4: Qualitative differences in the required blowing pressures anticipated for different types of fluids.